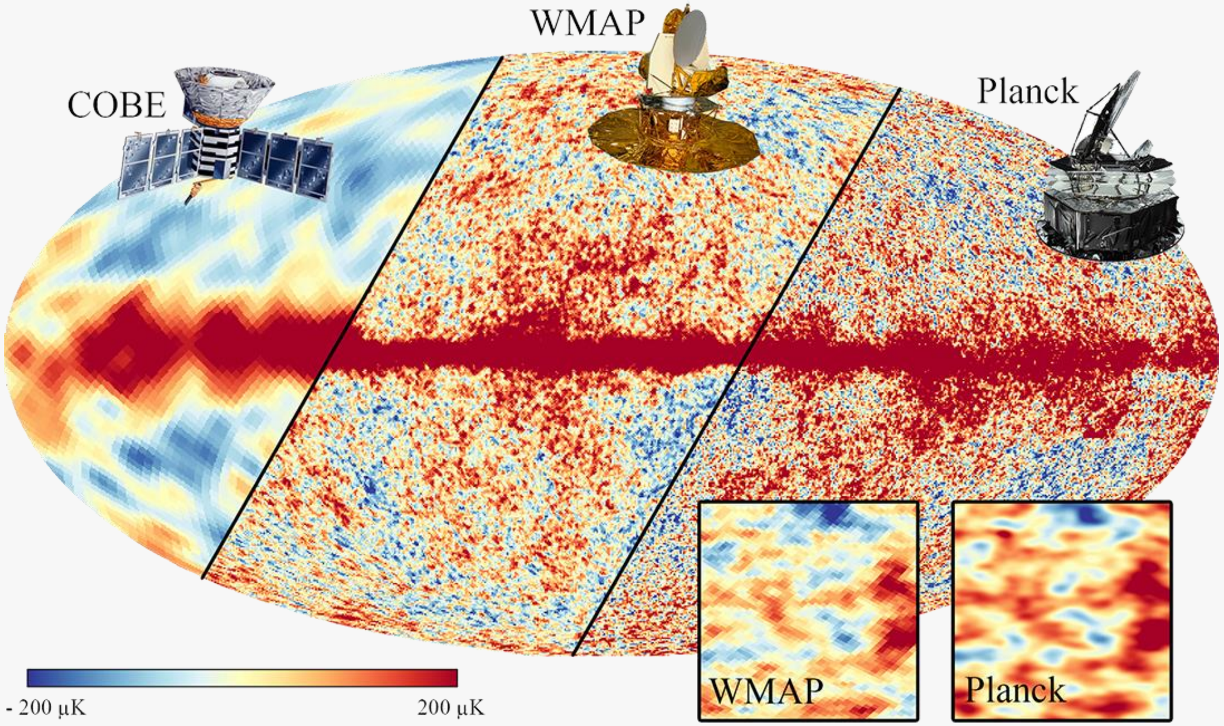
(INFLATIONARY) COSMOLOGY IN SCALAR–TENSOR THEORIES OF GRAVITATION



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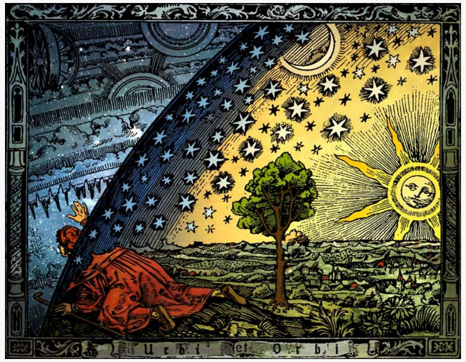
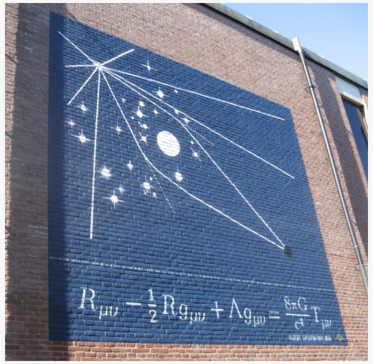
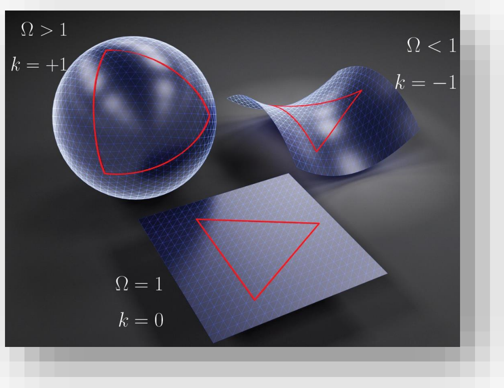
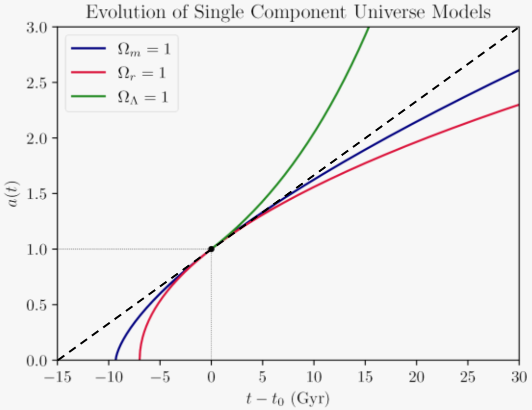
OUTLINE

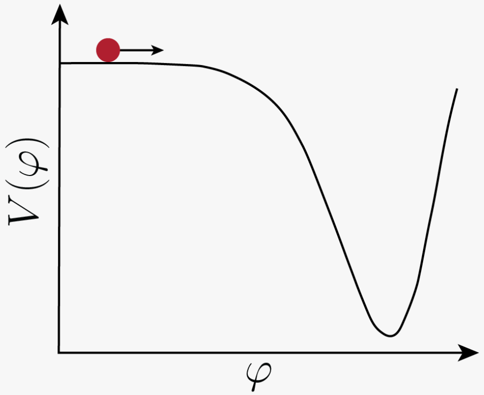
- A Brief Historical Introduction
- GR Elements of FRW Cosmology
- Dynamics of the Universe
- Some observations and coding
- Modifying Dynamics
- Inflationary Dynamics
- Inflationary Dynamics in a Modified Theory

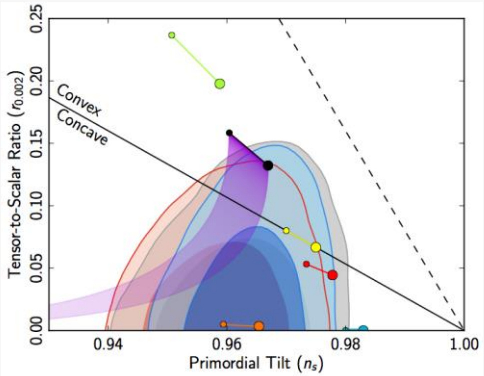






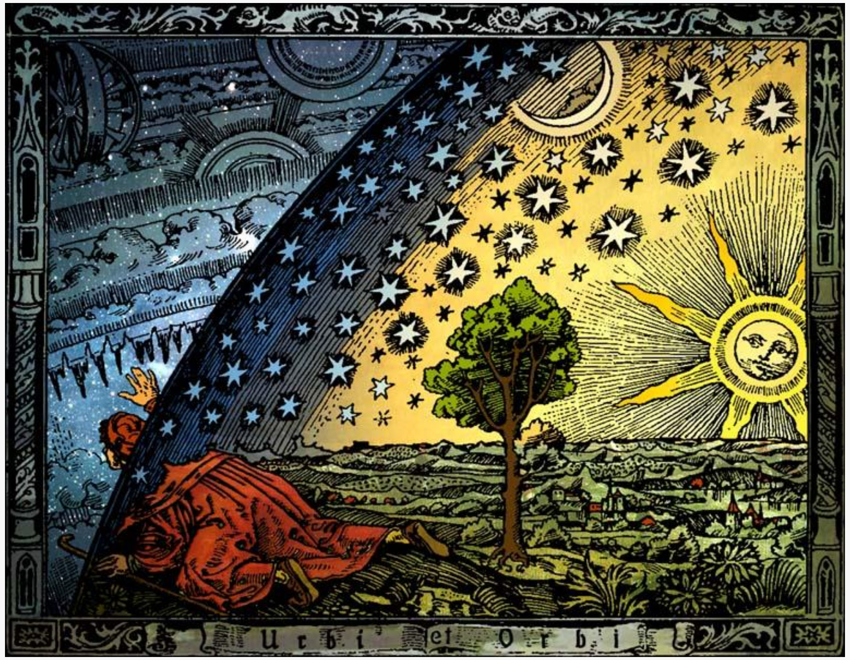






HISTORICAL INTRODUCTION

Where do we come from? What are we? Where are we going?



Colorized version of Flammarion engraving

Gauguin, 1897

HISTORICAL INTRODUCTION

Cosmology – study of the universe as a whole

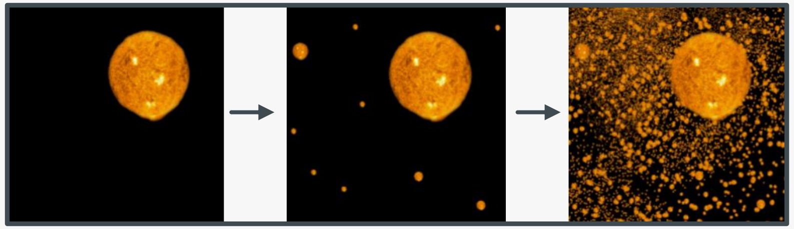
- Origin
- Evolution
- The ultimate “fate”

18th Century

– Catalogs of nebulas & first spectral analyses with no physical understanding

19th Century

– Olbers’ Paradox: Why the night sky is dark?



20th Century

– General Relativity (1915): Principle of General Covariance

Mass-energy tells spacetime how to curve, and curved ST tells mass-energy how to move

– Friedmann & Lemaitre (1920s): GR based expanding universe models, e.g. “Cosmic Egg”

– “Great Debate” btw Shapley & Curtis (1920s): Whether Milky Way is whole universe OR not?

HISTORICAL INTRODUCTION

Einstein & Hubble, @Mount Wilson Observatory, California (1931)

20th Century

– E. Hubble (1924): Andromeda (M31)

– E. Hubble (1929): Further galaxies receding faster – Expanding universe!

– Robertson – Walker (1930s): Metric of the curved spacetime

– Zwicky & Smith (1940s): Early mention of “Dark Matter”

– Gamow & Hermann (1948): Thermal radiation after “**hot Big Bang**”

– Penzias & Wilson (1964): Discovery of Cosmic Microwave Background

– Guth – Linde (1980-81): Inflation - accelerated expansion in the early stage of the universe

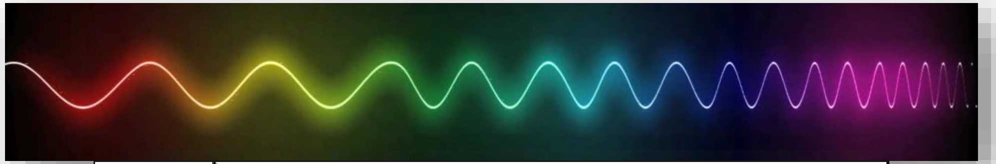
– Supernova Cosmology Project & High-Z Supernova Search Team(1998): Accelerated expansion of the universe

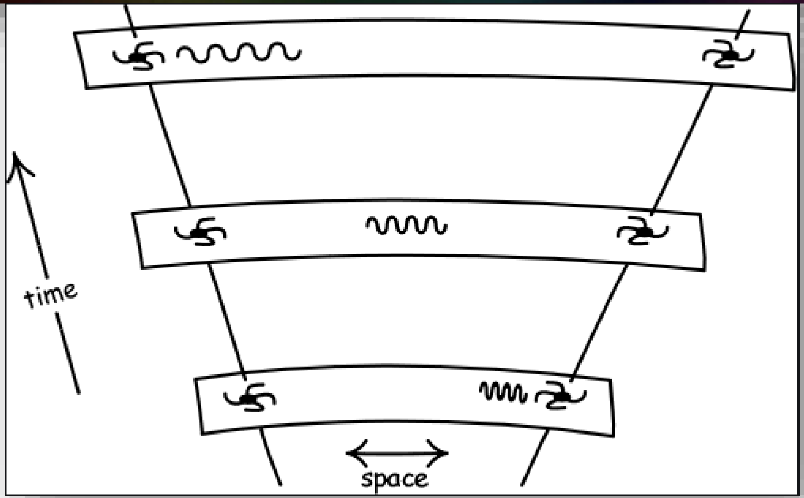
ASTRONOMICAL UNITS

Unit	In Metric Units	Galactic Scale
Distance	$1\ \text{Mpc} = 10^6\ \text{pc} = 3.1 \times 10^{22}\ \text{m}$	$R_{\text{gal}} = 50\,\text{kpc}$
Mass	$1M_\odot = 2 \times 10^{30}\ \text{kg}$	$M_{\text{gal}} = 10^{12} M_\odot$
Luminosity	$1L_\odot = 3.8 \times 10^{26}\ \text{watts}$	$L_{\text{gal}} = 3.6 \times 10^{10} L_\odot$
Time	$1\ \text{Gyr} = 10^9\ \text{year}$	

HUBBLE LAW AND EXPANDING UNIVERSE

Wavelength of photons gets “stretched” as the universe expands

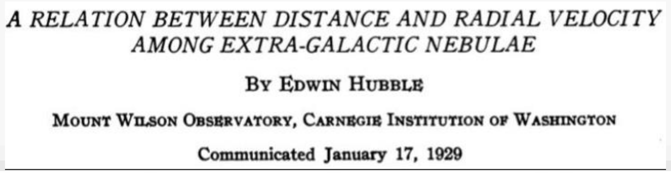




Redshift of the distant galaxies defined by

$$z \equiv \frac{\lambda_{\rm ob} - \lambda_{\rm em}}{\lambda_{\rm em}}$$

In 1929...


A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY AMONG EXTRA-GALACTIC NEBULAE

By Edwin Hubble

Mount Wilson Observatory, Carnegie Institution of Washington

Communicated January 17, 1929

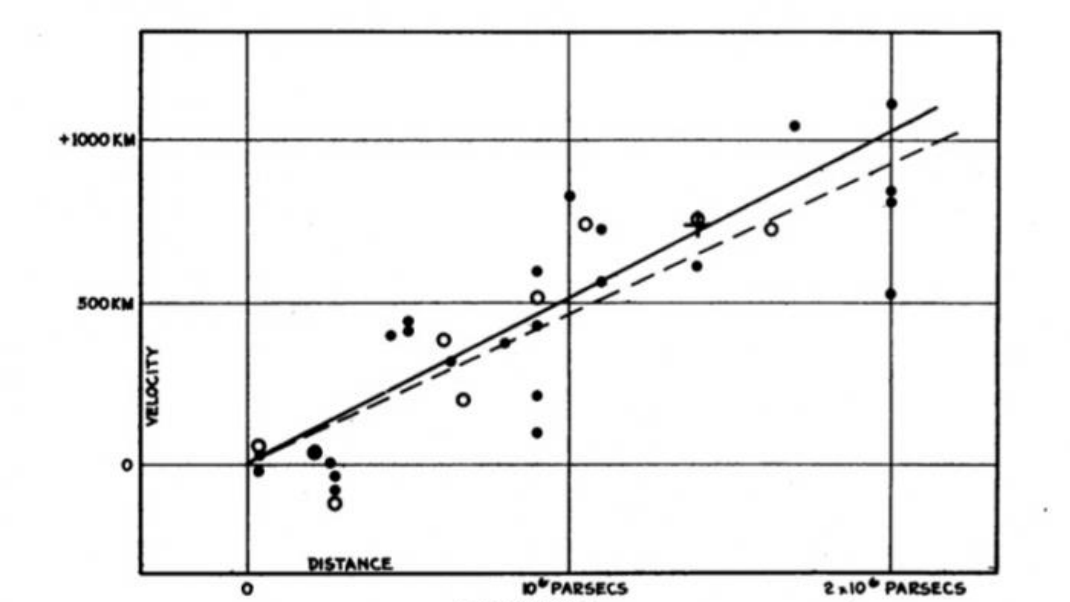


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Hubble Law

$v = H_0\, d$ → Further galaxies receding faster

↓

Hubble constant (parameter)

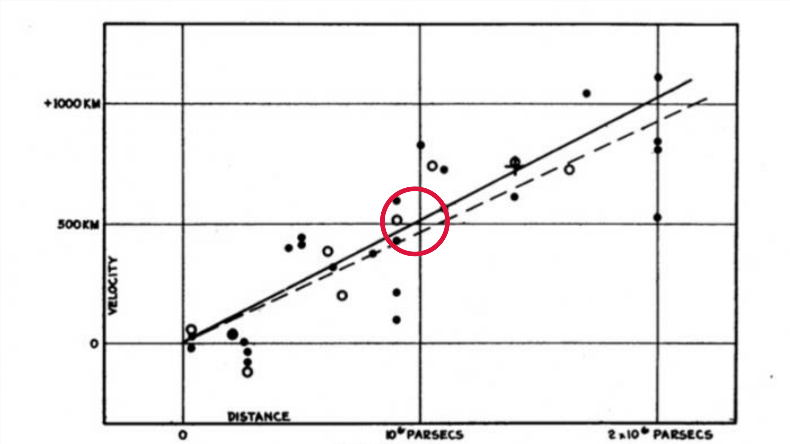


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Hubble Law

$$v = H_0 \, d$$

$$[H] = \left[\frac{1}{T}\right] \rightarrow \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

$\longrightarrow$

According to the plot

$$H_0 = 500\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Modern observations

$$H_0 \approx 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Hubble Time (t_H)

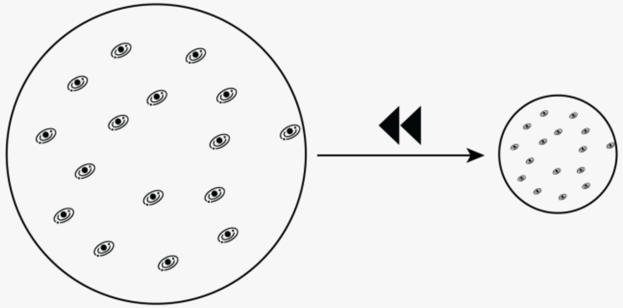
$$\frac{1}{H_0} = 1.95\,\mathrm{Gyr}$$

$$\frac{1}{H_0} = 13.97\,\mathrm{Gyr}$$

If the expansion had been linear, i.e. constant Hubble Parameter

Run back the film to “Hubble time” ago..

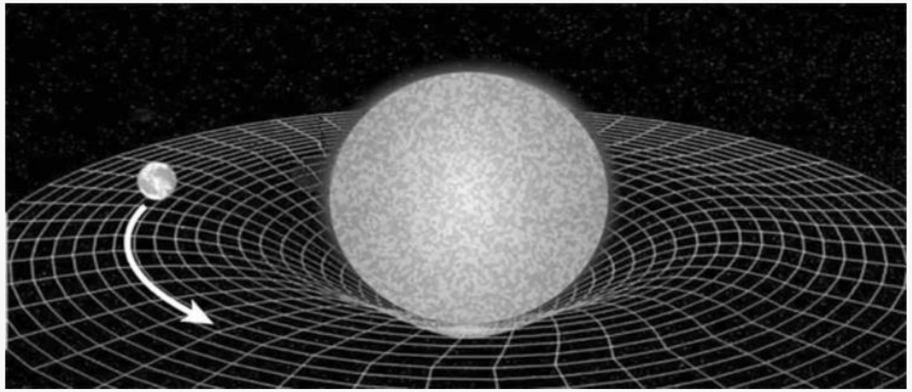
Galaxies were crammed together into a small volume



DYNAMICS OF THE UNIVERSE

Framework: General Relativity

$$G_{\mu\nu} \equiv \underbrace{R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu}}_{\text{Geometry}} = \underbrace{\frac{8\pi G}{c^4}T_{\mu\nu}}_{\text{EM Content}}$$



Gravity is the manifestation of curvature!

$g_{\mu\nu}$: Metric

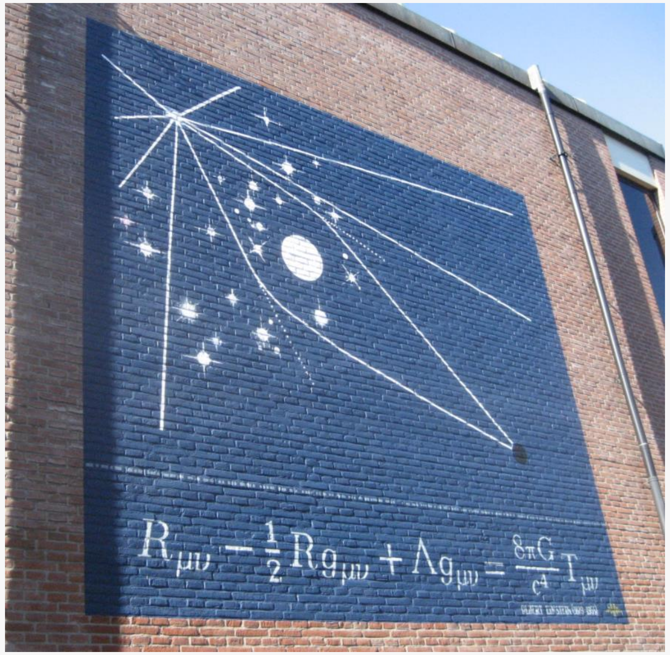
${R^{\lambda}}_{\mu\nu\kappa}$: Riemann Tensor

$R_{\mu\nu}$: Ricci tensor

R : Ricci scalar

${\Gamma^{\lambda}}_{\mu\nu}$: Christoffel Symbol

$T_{\mu\nu}$: Energy − Momentum Tensor



Leiden, Netherlands

METRIC

Metric describes the structure of spacetime and provides a local measure of invariant distance

$$x^\mu \to (t, x, y, z)$$

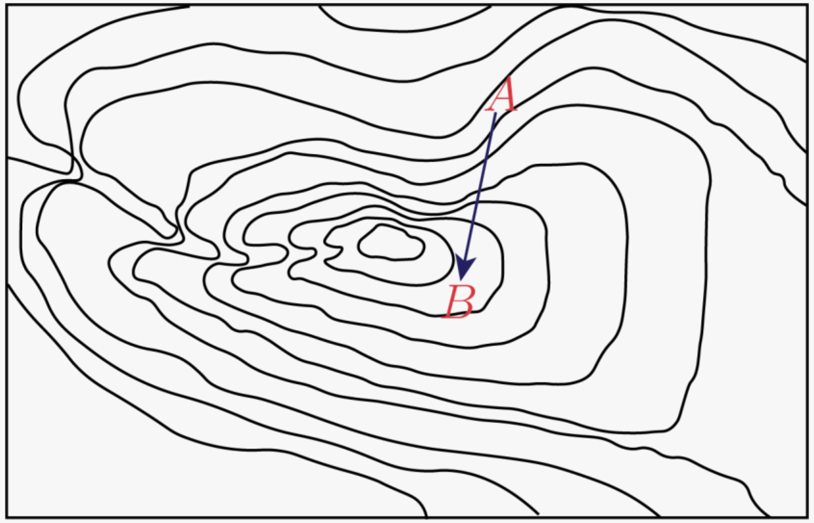
$$ds^2 = dx^2 + dy^2$$

$$ds^2 = dr^2 + r^2 d\theta^2$$

$$ds^2 = \sum_{i,j=1,2} g_{ij}\, dx^i\, dx^j$$

$$x^1 = x,\ x^2 = y,\ g_{ij} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x^1 = r,\ x^2 = \theta,\ g_{ij} = \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}$$



→ The actual physical distance depends on **“topography”**

METRIC

In classical Newtonian Mechanics, gravity is an external force, and particles move in a gravitational field.

In GR, gravity is encoded within the **metric** and the particles move in the curved space.

Using *Einstein summation convention* the line element can be written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \xrightarrow{\text{in 4D}} \mu, \nu = 0, 1, 2, 3$$

FRW Metric

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \longrightarrow \quad g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & a^2 r^2 & 0 \\ 0 & 0 & 0 & a^2 r^2 \sin^2\theta \end{bmatrix}$$

Scale Factor: shows how universe expands/contradicts with time

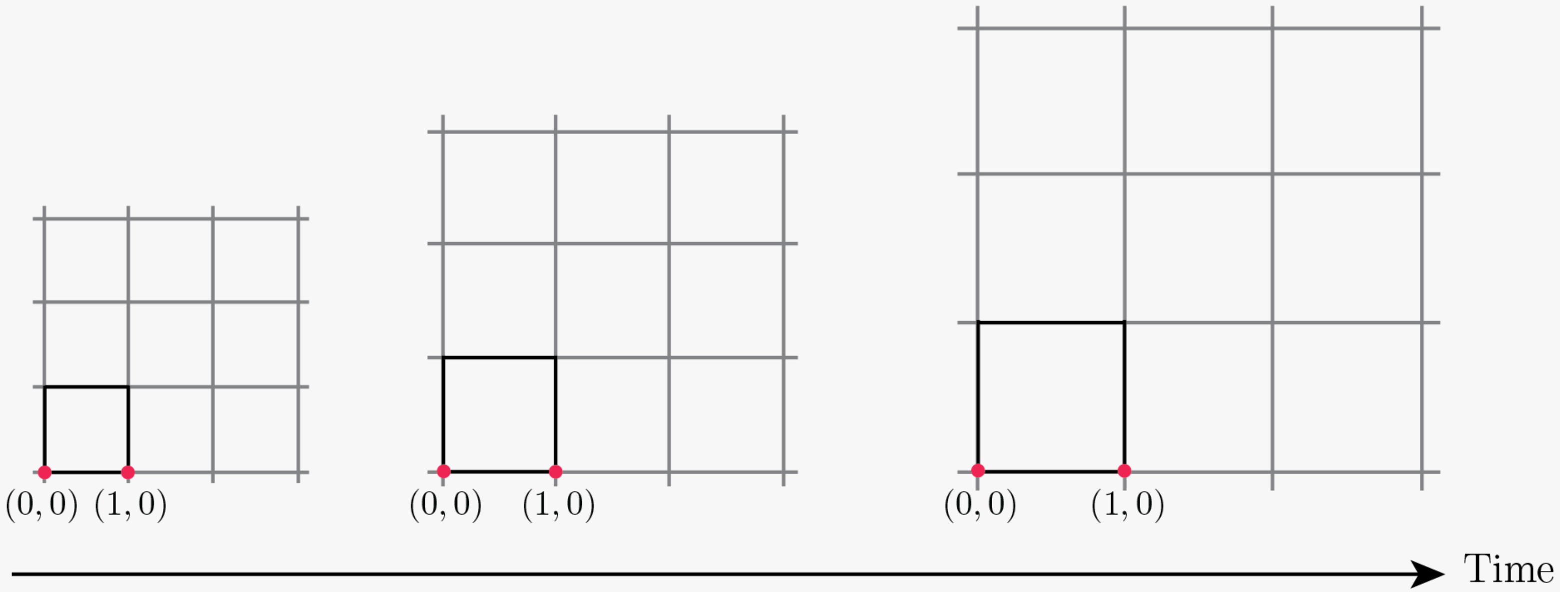
Curvature Constant (k): can take the values -1, 0, +1 corresponding to the geometry of space

METRIC

FRW Metric

$$ds^2 = -dt^2 + \boxed{a^2(t)}\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

Scale Factor: shows how universe expands/contradicts with time



The comoving distance between points on a coordinate grid remains constant as the universe expands.

The physical distance is proportional to the comoving distance times the scale factor $a(t)$

GR ELEMENTS OF FRW COSMOLOGY - I

$$\Gamma^{\lambda}{}_{\mu\nu} \equiv \frac{1}{2} g^{\lambda\sigma} (\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu})$$

$$R^{\lambda}{}_{\mu\nu\kappa} \equiv \partial_\nu \Gamma^{\lambda}{}_{\mu\kappa} - \partial_\kappa \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\eta}{}_{\mu\nu} \Gamma^{\lambda}{}_{\kappa\eta} + \Gamma^{\eta}{}_{\mu\kappa} \Gamma^{\lambda}{}_{\nu\eta}$$

$$R_{\mu\nu} \equiv R^{\lambda}{}_{\mu\lambda\nu}$$

$$R \equiv g^{\mu\nu} R_{\mu\nu}$$

GR ELEMENTS OF FRW COSMOLOGY - II

$$R_{00} = R^{\lambda}{}_{0\lambda 0} = -3\frac{\ddot{a}}{ac^2}$$

$$R_{11} = R^{\lambda}{}_{1\lambda 1} = \frac{a\ddot{a} + 2\dot{a}^2 + 2kc^2}{c^2(1 - kr^2)}$$

$$R_{22} = R^{\lambda}{}_{2\lambda 2} = \frac{r^2}{c^2}(a\ddot{a} + 2\dot{a}^2 + 2kc^2)$$

$$R_{33} = R^{\lambda}{}_{3\lambda 3} = \frac{r^2 \sin^2\theta}{c^2}(a\ddot{a} + 2\dot{a}^2 + 2kc^2)$$

$$R = g^{\mu\nu}R_{\mu\nu} = 6\left[\frac{\ddot{a}}{ac^2} + \left(\frac{\dot{a}}{ac}\right)^2 + \frac{k}{a^2}\right]$$

DYNAMICS OF THE UNIVERSE

Dynamics of the universe is governed by following set of equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{kc^2}{a^2(t)}$$

$$\frac{2\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{8\pi G}{c^2}p - \frac{kc^2}{a^2(t)}$$

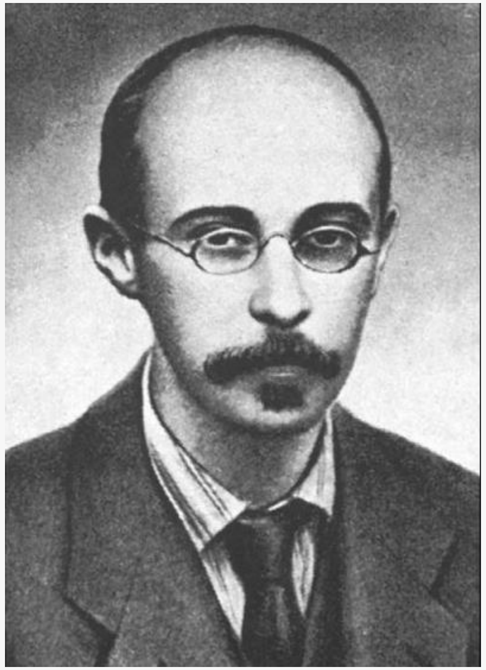
→ Friedmann Equations

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

→ Fluid (continuity) Equation

$$p = \omega\rho$$

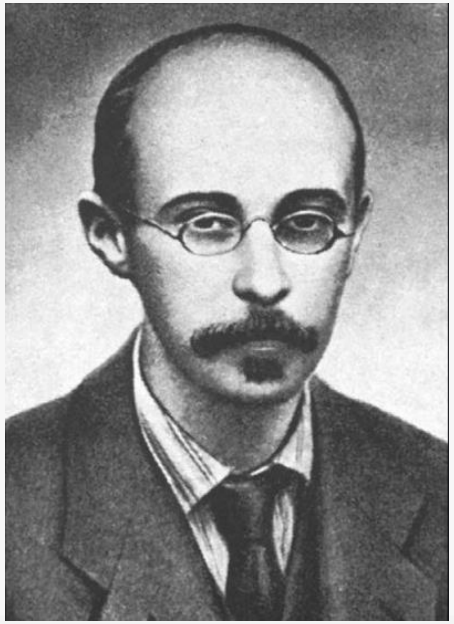
→ Equation of State (EoS)



DYNAMICS OF THE UNIVERSE

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{kc^2}{a^2(t)}$$

1st Friedmann Equation



Defining LHS as Hubble Parameter

$$H \equiv \frac{\dot{a}}{a} \implies H^2(t) = \frac{8\pi G}{3c^2}\rho(t) - \frac{kc^2}{a^2(t)}$$

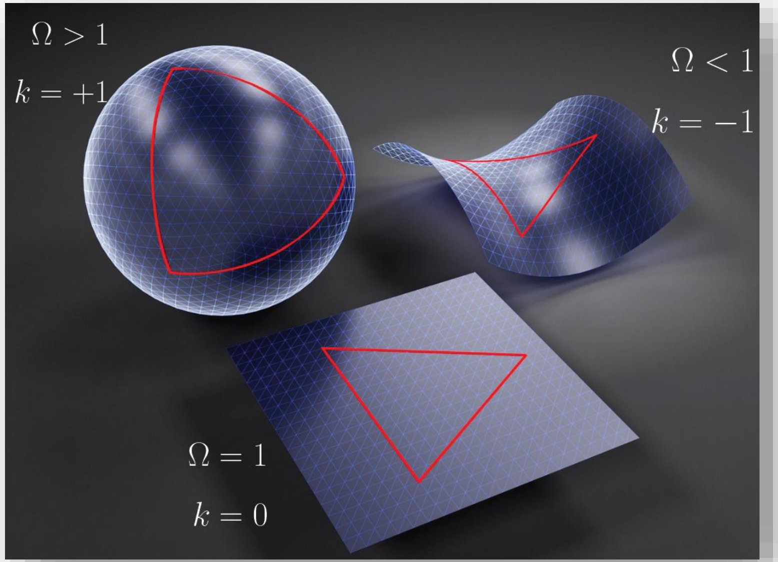
Spatially flat, $k = 0$

$$H^2 = \frac{8\pi G}{3c^2}\rho$$

"Critical Density"

For a given Hubble parameter, we can define "critical density" as

$$H^2 = \frac{8\pi G}{3c^2}\rho_c \implies \rho_c = \frac{3c^2}{8\pi G}H^2$$



DYNAMICS OF THE UNIVERSE

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{kc^2}{a^2(t)}$$ 1st Friedmann Equation

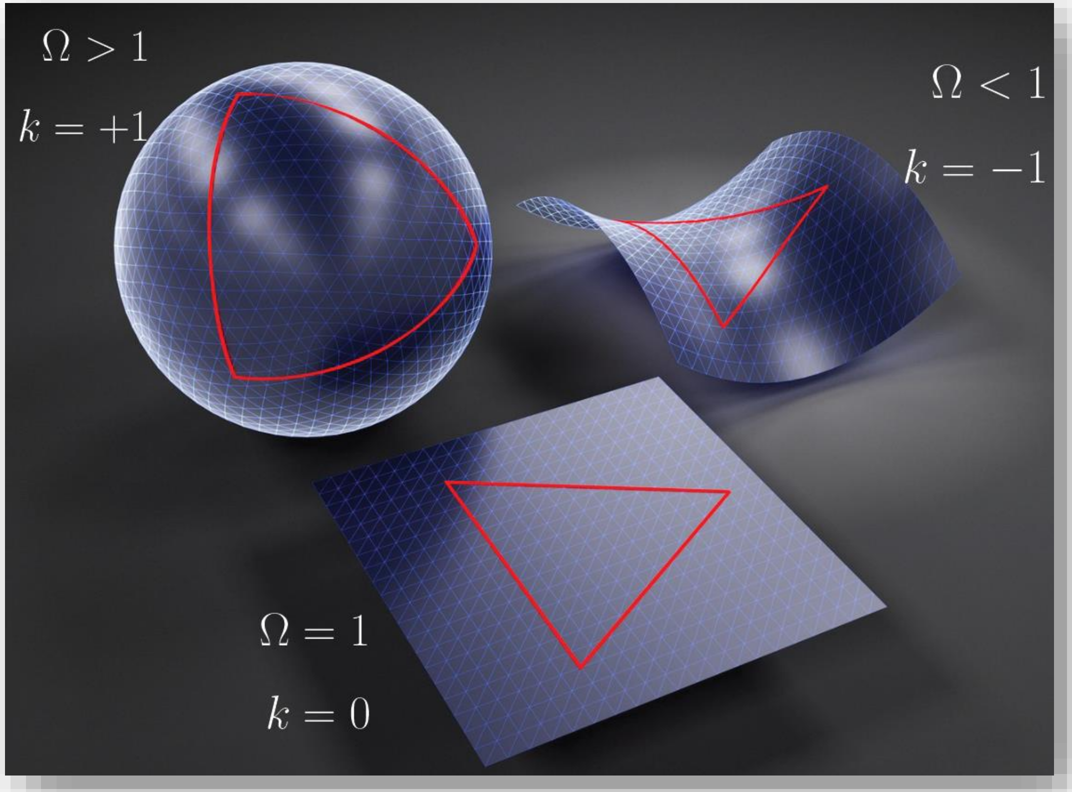
$$H^2 = \frac{8\pi G}{3c^2}\rho_c \implies \rho_c = \frac{3c^2}{8\pi G}H^2$$ — "Critical Density"

Define dimensionless "density parameter"

$$\Omega = \frac{\rho}{\rho_c} \implies \boxed{1 - \Omega(t) = -\frac{kc^2}{a^2(t)H^2(t)}}$$

Directly indicates the geometry

$$\begin{cases} \Omega > 1 \rightarrow k = +1 \\ \Omega = 1 \rightarrow k = 0 \\ \Omega < 1 \rightarrow k = -1 \end{cases}$$



According to current observations, geometry of the universe is close to be flat

EVOLUTION OF THE UNIVERSE

Using fluid equation, let us determine the evolution of energy densities

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + \omega\rho)$$

$$\int \frac{\dot{\rho}}{\rho} = -3(\omega + 1)\int \frac{\dot{a}}{a}$$

$$\rho = \rho_0\, a^{-3(\omega+1)}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3c^2}(\rho + 3p)$$

$$\ddot{a} > 0$$

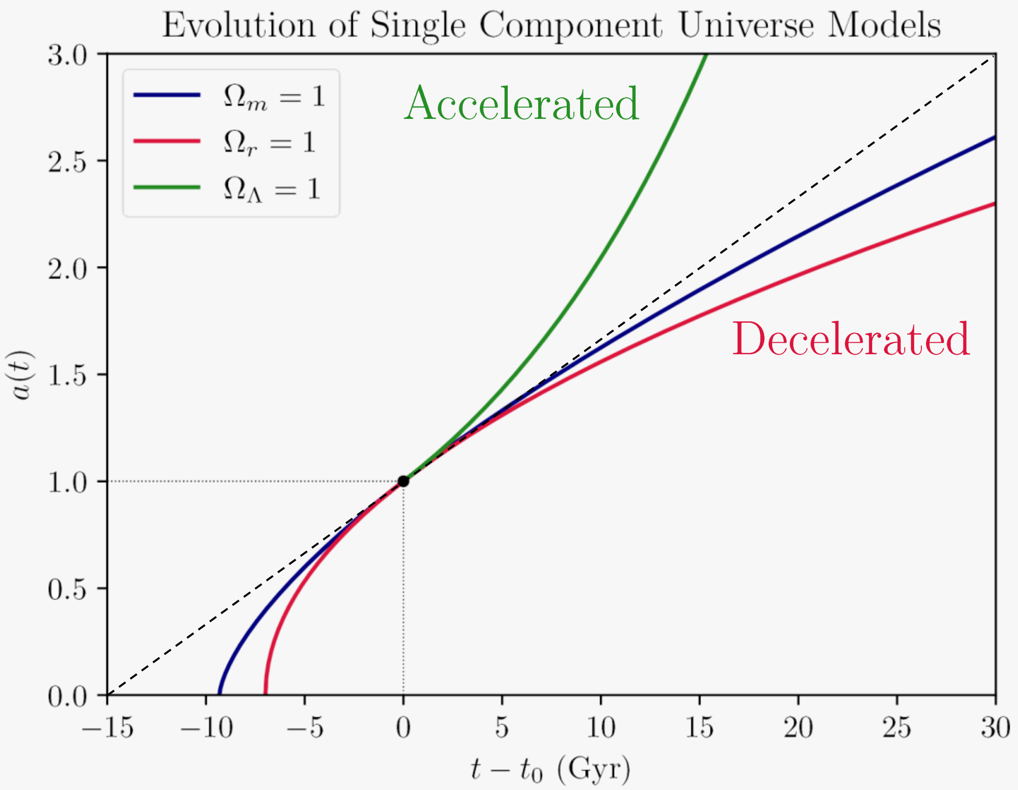
$$\boxed{\omega < -\frac{1}{3}}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_0}{3c^2}a^{-3(1+\omega)}$$

Educated guess: $a \propto t^\gamma$

$$a \propto t^{\,2/3(1+\omega)}$$

Matter	Radiation	Dark Energy
$\omega_\mathrm{m} = 0$	$\omega_\mathrm{r} = 1/3$	$\omega_\Lambda = -1$
$a \propto t^{2/3}$	$a \propto t^{1/2}$	$a \propto e^t$
$\rho_\mathrm{m} = \rho_{\mathrm{m},0}\, a^{-3}$	$\rho_\mathrm{r} = \rho_{\mathrm{r},0}\, a^{-4}$	$\rho_\Lambda = \rho_{\Lambda,0}$



In terms of dimensionless density parameter, Friedmann Equation can be written as

$$\frac{H^2(t)}{H_0^2} = \frac{\Omega_{\mathrm{r},0}}{a^4} + \frac{\Omega_{\mathrm{m},0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

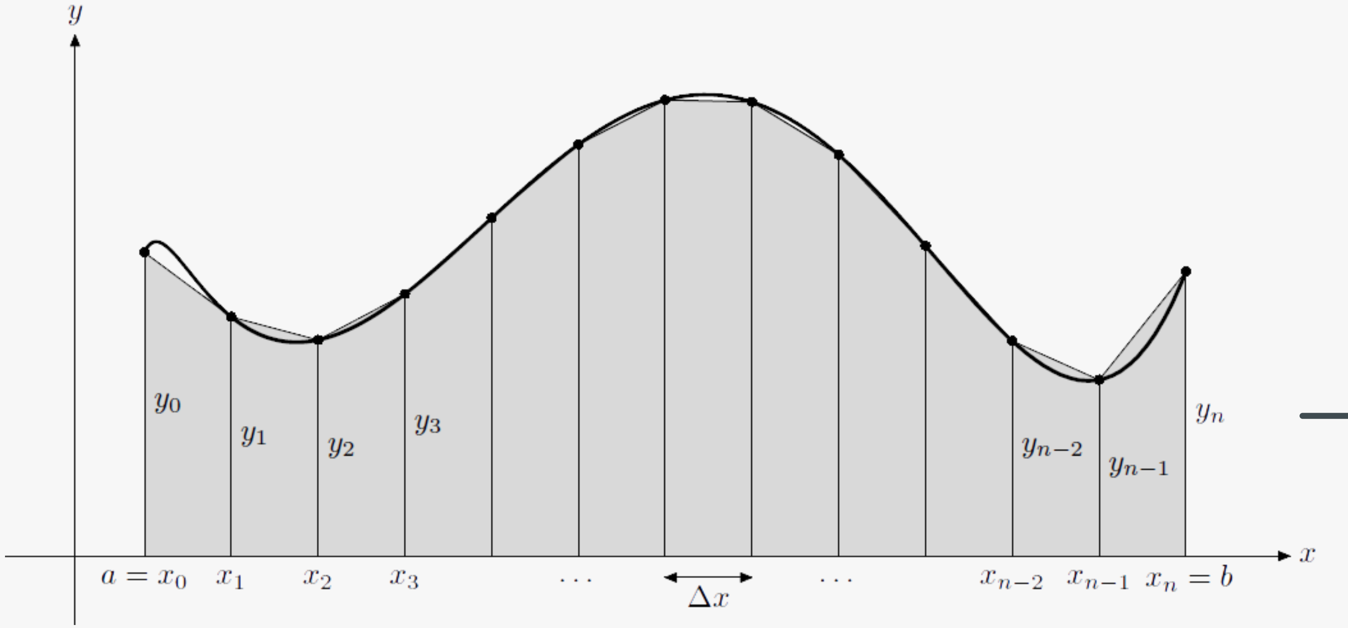
$$\frac{da}{dt} = H_0 \left(\frac{\Omega_{\mathrm{r},0}}{a^2} + \frac{\Omega_{\mathrm{m},0}}{a} + a^2\,\Omega_{\Lambda,0} + 1 - \Omega_0 \right)^{\frac{1}{2}}$$

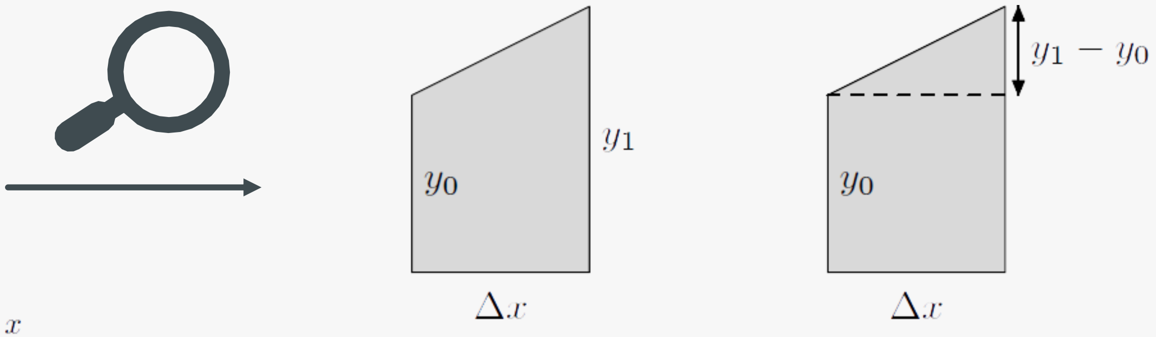
$$H_0(t_0 - t) = \int_0^a \frac{da'}{\left(\Omega_{\mathrm{r},0}/a'^2 + \Omega_{\mathrm{m},0}/a' + a'^2\,\Omega_{\Lambda,0} + 1 - \Omega_0\right)^{1/2}}$$

Can be integrated (analytically) under some conditions, however in general requires numerical integration.

⚠ The ingredients of such a universe models do **not** interact with each other

NUMERICAL QUADRATURE - TRAPEZOID





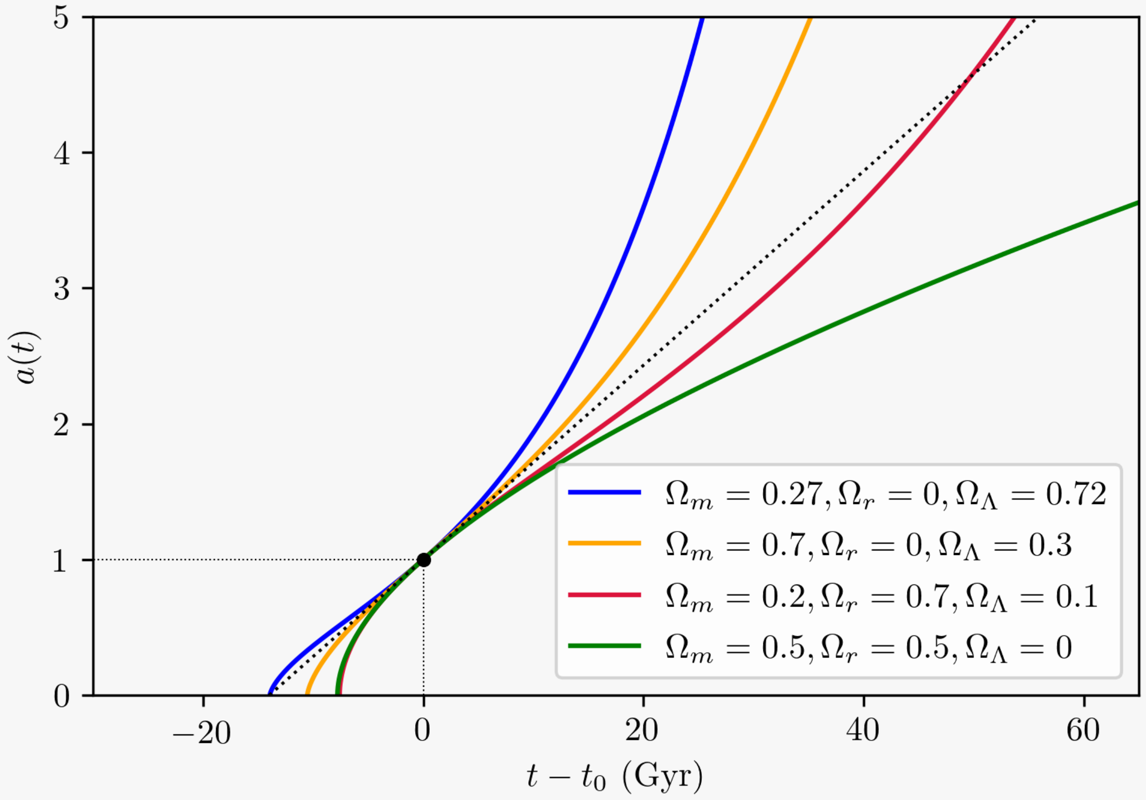
$$A = y_0\Delta x + \frac{1}{2}(y_1 - y_0)\Delta x = \frac{(y_0 + y_1)\Delta x}{2}$$

$$\int_a^b f(x)dx \approx \frac{(y_0 + y_1)\Delta x}{2} + \frac{(y_1 + y_2)\Delta x}{2} + \frac{(y_2 + y_3)\Delta x}{2} + \cdots$$

$$\boxed{\int_a^b f(x)dx \approx \frac{\Delta x}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)}$$

For complicated equations with smaller step-size use other methods, e.g. Gaussian quadrature.

EVOLUTION OF UNIVERSE AND THE CONTENT




```
def Friedmann(a,m,r,l):
    Omega_0 = m + r + l
    t = (1/ np.sqrt((r/(a*a)) + (m/a) + (l * (a*a)) + (1-Omega_0)))/H_0

    return t

def Trapezoidal(a,b,m,r,l):
    n = 10000                    # Step number
    deltaX = (b-a)/n             # Step size

    AGE = 0

    x = np.zeros(n)
    y = np.zeros(n)
    z = np.zeros(n)

    for i in range(n):
        x[i] = a + i*deltaX # Increment
        y[i] = Friedmann(x[i],m,r,l) # Numerical value of the function
        z[i] = (deltaX/2) * (2*np.sum(y) - y[0] - y[n-1]) # Total integration result

        if (x[i] == 1 or 1-eps <= x[i] <= 1+eps):
            AGE = z[i]

    print ('Age of the universe with m = %5.3f, r = %5.3f, lambda = %5.3f is %5.3f G
yr' %(m,r,l,AGE))

    return x,z,AGE
```

TWO COMPLIMENTARY OBSERVATIONS

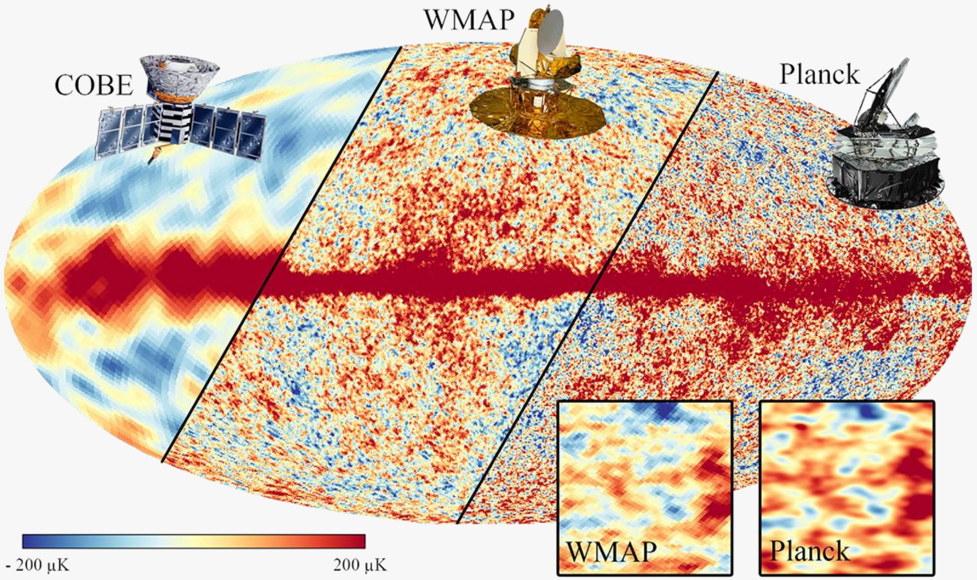
So, how do we know the accelerated expansion?

How do we constrain the density parameters discussed in previous section?

Accelerated Expansion

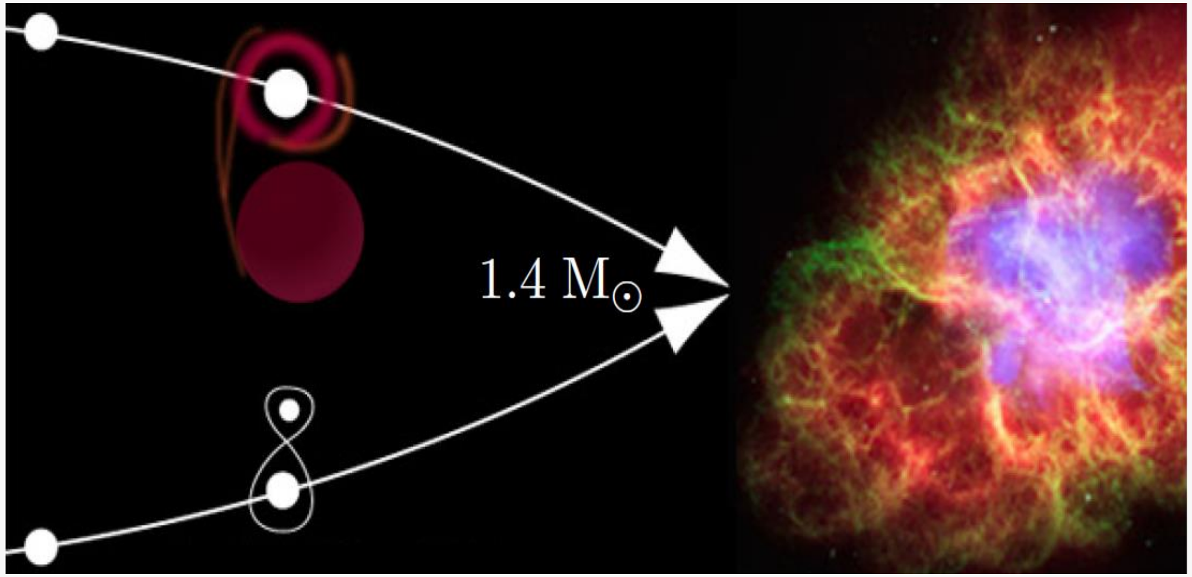
Type Ia Supernovae Observations

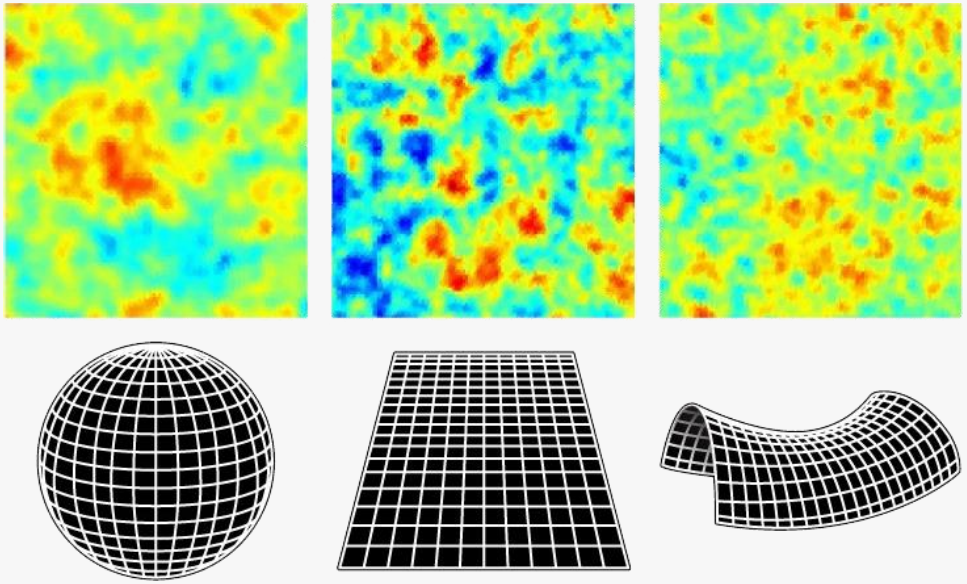
Flat Geometry



CMB Anisotropies

TWO COMPLIMENTARY OBSERVATIONS





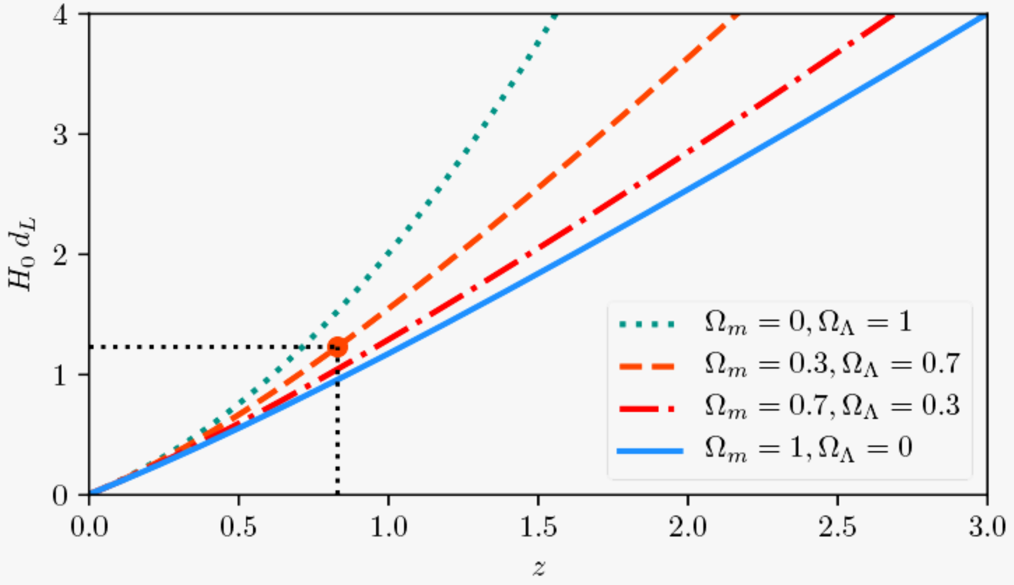
$$F = \frac{L}{4\pi d_L^2}$$

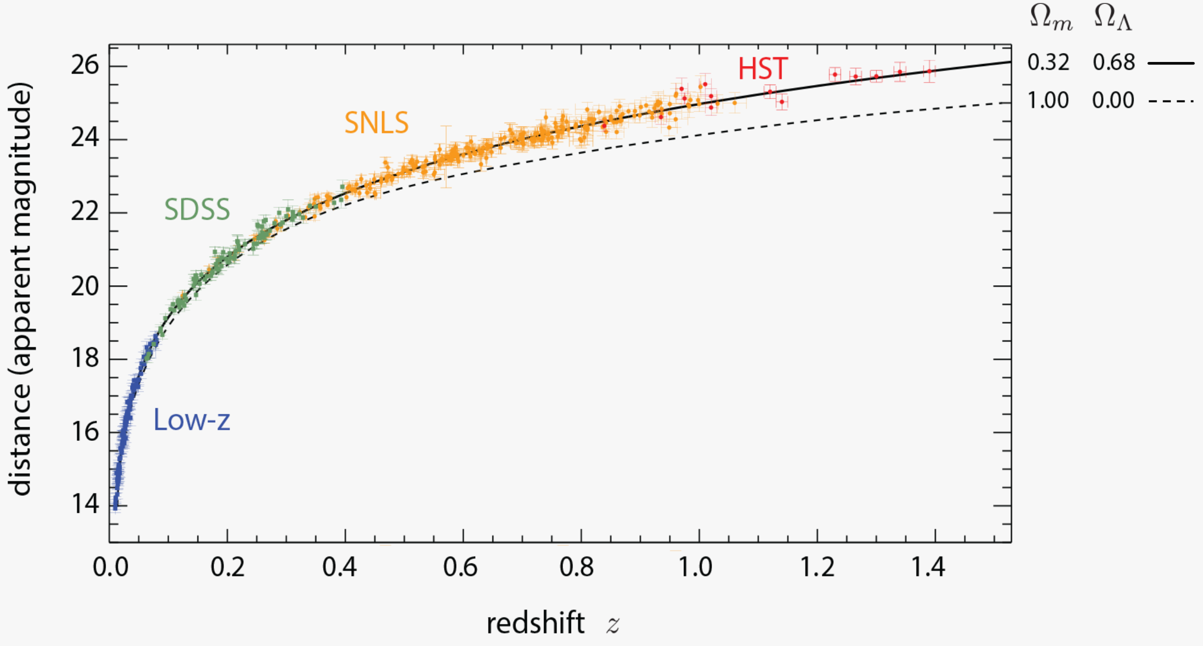
$$\boxed{d_L = \frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{\sum_i \Omega_{i,0}(1+z')^{3(1+\omega_i)}}}}$$

$$\frac{\delta T}{T}(\theta,\phi) \equiv \frac{T(\theta,\phi) - \langle T\rangle}{\langle T\rangle}$$

$$C(\theta) = \left\langle \frac{\delta T}{T}(\hat{n})\frac{\delta T}{T}(\hat{n}')\right\rangle$$

TWO COMPLIMENTARY OBSERVATIONS

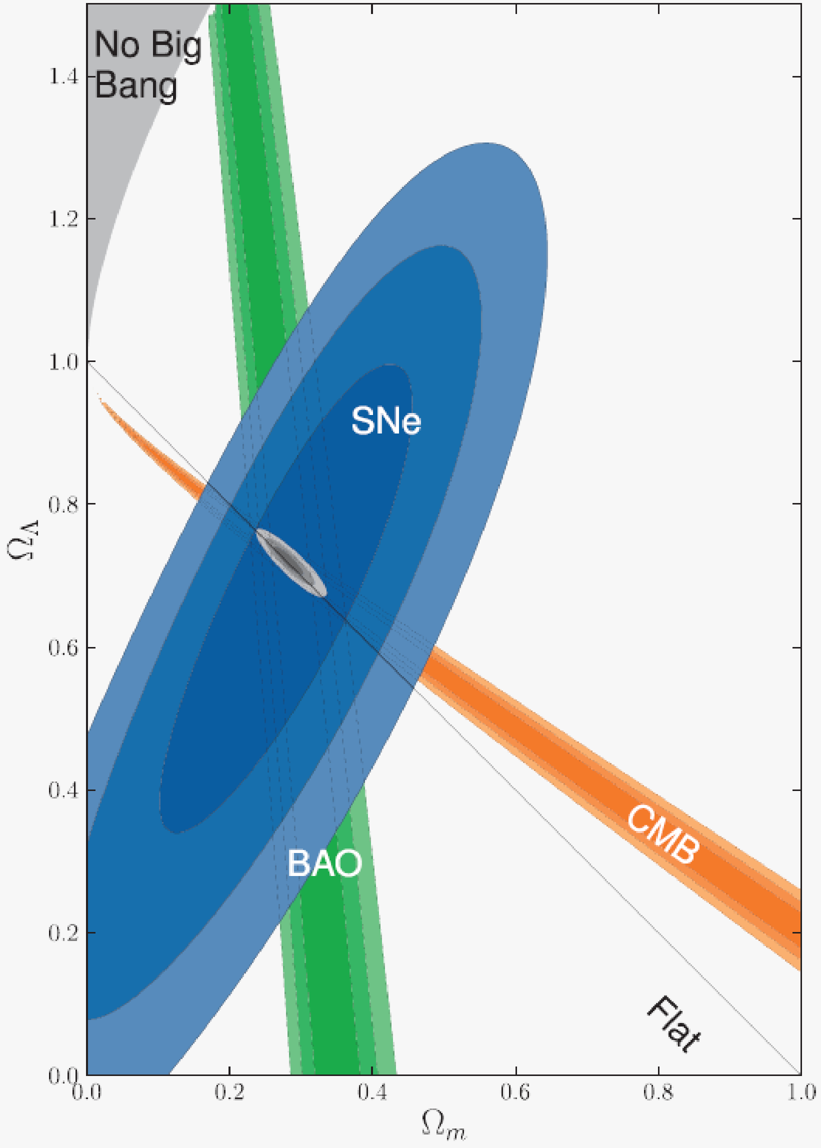




Best Fit

$\Omega_m \approx 0.3$

$\Omega_\Lambda \approx 0.7$



BUT...

Cosmological constant is interpreted as the “vacuum energy”.

“The” Cosmological Constant Problem: There is a huge difference btw observed and expected energy density (about 10^{120} orders of magnitude)

Cosmic Coincidence (“why now”) Problem: Why ρ_Λ has the same order of magnitude with the present energy density of “matter”.

Why does cosmic acceleration happen to begin now, not in the past or future?

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Possible Modifications

a) RHS: Energy – Matter Content

b) LHS: Gravity Theory

MODIFYING THEORY – LAGRANGIAN FORMALISM

GR Action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + \mathcal{L}_M \right] \xrightarrow{\delta S = 0} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$T_{\mu\nu} \equiv \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}} = -2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}} + g_{\mu\nu} \mathcal{L}_m$$

$$S = \int \mathrm{d}^4x \sqrt{|g|} \left\{ R - \frac{1}{2} g^{\mu\nu} \left(\partial_\mu \varphi\right) \left(\partial_\nu \varphi\right) - V(\varphi) \right\}$$

FRW →

$$3H^2 = \kappa^2 \left(\frac{\dot{\varphi}^2}{2} + V \right) \quad , \quad 2\dot{H} = \kappa^2 \dot{\varphi}^2$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V(\varphi)' = 0$$

Einstein Gravity

$$S = \int \mathrm{d}^4x \sqrt{|g|} \left\{ F(\varphi) R - \frac{1}{2} g^{\mu\nu} \left(\partial_\mu \varphi\right) \left(\partial_\nu \varphi\right) - V(\varphi) \right\} \quad \text{where } F(\varphi) = \frac{1}{2} \left(\frac{1}{\kappa^2} + \xi \varphi^2 \right)$$

NMC scalar field

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{2F(\varphi)} \left\{ -g_{\mu\nu} \left(\frac{1}{2} (\partial\varphi)^2 + V(\varphi) \right) + 2\nabla_\mu \nabla_\nu F(\varphi) - 2 g_{\mu\nu} \Box F(\varphi) + \partial_\mu \varphi \partial_\nu \varphi \right\}$$

MINIMALLY COUPLED INFLATON

$$S = \int \mathrm{d}^4x \sqrt{|g|} \left\{ R - \frac{1}{2} g^{\mu\nu} \left(\partial_\mu \varphi\right) \left(\partial_\nu \varphi\right) - V(\varphi) \right\}$$

$\xrightarrow{\delta/\delta g^{\mu\nu}}$ $3H^2 = \kappa^2 \left(\frac{\dot{\varphi}^2}{2} + V \right) \quad , \quad 2\dot{H} = \kappa^2 \dot{\varphi}^2$

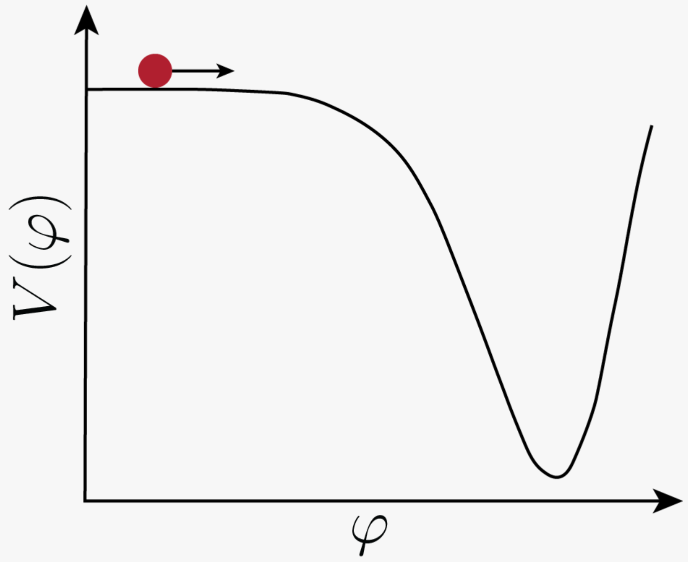
$\xrightarrow{\delta/\delta \varphi}$ $\ddot{\varphi} + 3H\dot{\varphi} + V(\varphi)' = 0$

Can such a field result in accelerated expansion (or *inflation*)?

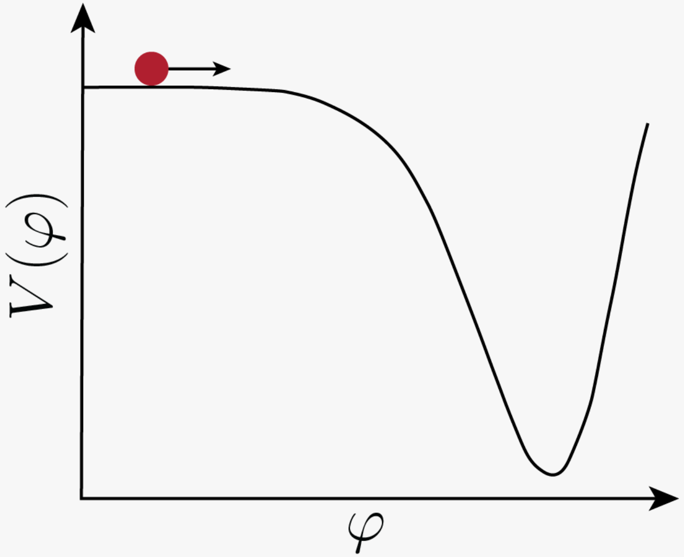
Answer: Under so-called slow roll conditions, yes! ✓

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \; p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$\boxed{\omega_\varphi = \frac{p}{\rho} = \frac{\dot{\varphi}^{\,2} - 2V(\varphi)}{\dot{\varphi}^{\,2} + 2V(\varphi)}} \xrightarrow{V(\varphi) \gg \dot{\varphi}^2} \omega_\varphi \simeq -1$$



SLOW-ROLL EQUATIONS

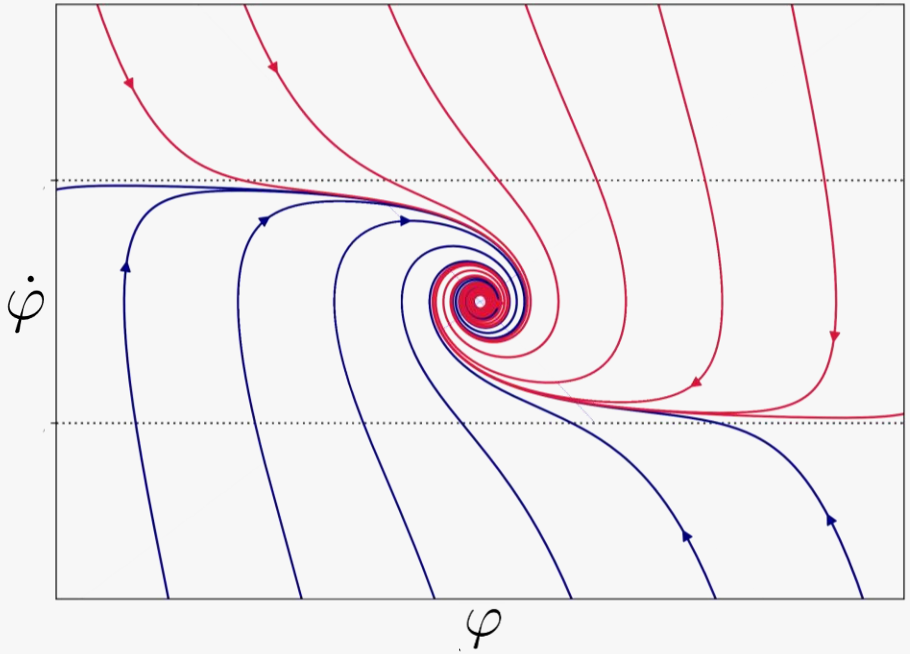


Friedmann equation

$$H^2 = \kappa^2 \left[\frac{\dot{\varphi}^2}{2} + V(\varphi) \right] \xrightarrow{\text{SR}} H^2 = \kappa^2 V(\varphi)$$

EoM

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV(\varphi)}{dt} = 0 \xrightarrow{\text{SR}} 3H\dot{\varphi} = -V'$$



RELATING DYNAMICS WITH PARAMETERS

Hubble SR Parameters

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2}$$

$$\eta_H \equiv \frac{\ddot{H}}{H\,\dot{H}}$$

Potential SR Parameters

$$\epsilon_V = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V = \frac{1}{\kappa^2}\frac{V''}{V}$$

As long as universe expands exponentially, these parameters <<1

End of inflation: $\epsilon = 1$

Number of e-folding: $N \equiv \ln\frac{a_f}{a_i} = \int_{t_*}^{t_e} H dt$

RELATING DYNAMICS WITH PARAMETERS - II

Potential SR Parameters

$$\epsilon_V = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V = \frac{1}{\kappa^2}\frac{V''}{V}$$

End of inflation: $\epsilon = 1$

Number of e-folding: $N \equiv \ln\frac{a_f}{a_i} = \int_{t_*}^{t_e} H dt$

$$H^2 \simeq \kappa^2 V(\varphi)$$

$$3H\dot{\varphi} = -V'$$

Let us express e-folding in terms of field itself and SR parameters..

$$N = \int H dt = \int H \frac{dt}{d\varphi} d\varphi = \int_{\varphi_*}^{\varphi_e} \frac{H}{\dot{\varphi}} d\varphi \xrightarrow{3H\dot{\varphi} = -V'} N = \int_{\varphi_*}^{\varphi_e} \frac{3H^2}{V'} d\varphi = -\kappa^2 \int_{\varphi_*}^{\varphi_e} \frac{V}{V'} d\varphi$$

$$\boxed{N = -\kappa \int_{\varphi_*}^{\varphi_e} \frac{d\varphi}{\sqrt{2\epsilon_\varphi}}}$$

TOY MODEL: MONOMIAL POTENTIAL

$$N = -\kappa^2 \int_{\phi_*}^{\phi_e} \frac{V}{V'} d\varphi \qquad \epsilon_V = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V = \frac{1}{\kappa^2}\frac{V''}{V}$$

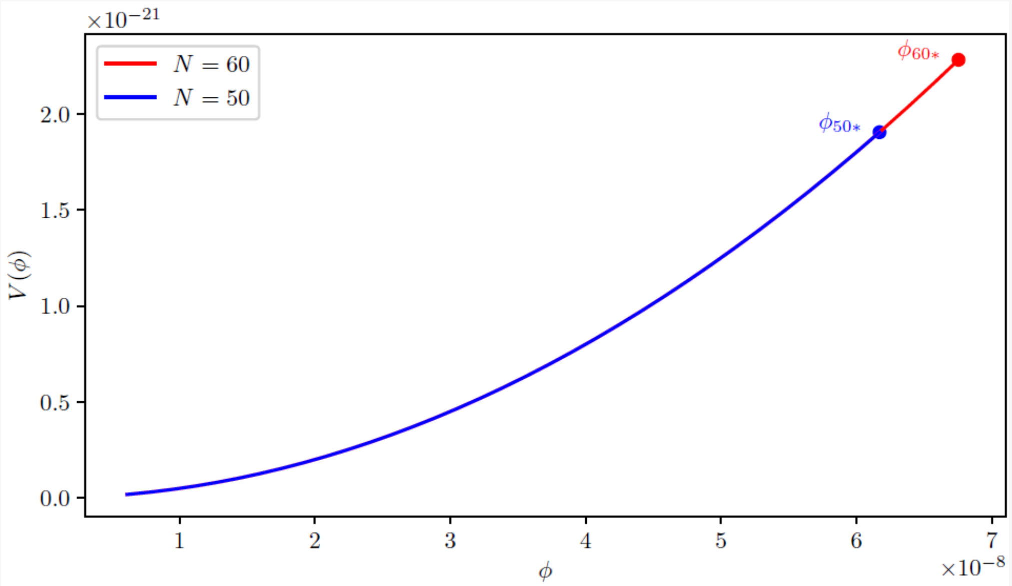
$$V(\varphi) \propto \varphi^n \qquad V' = n\varphi^{n-1} \quad V'' = n(n-1)\varphi^{n-2}$$

$$\epsilon = \frac{1}{2\kappa^2}\left(\frac{n\varphi^{n-1}}{\varphi^n}\right)^2 = \frac{1}{2\kappa^2}\frac{n^2}{\varphi^2} \xrightarrow{\epsilon = 1} \boxed{\varphi_e^2 = \frac{n^2}{2\kappa^2}}$$

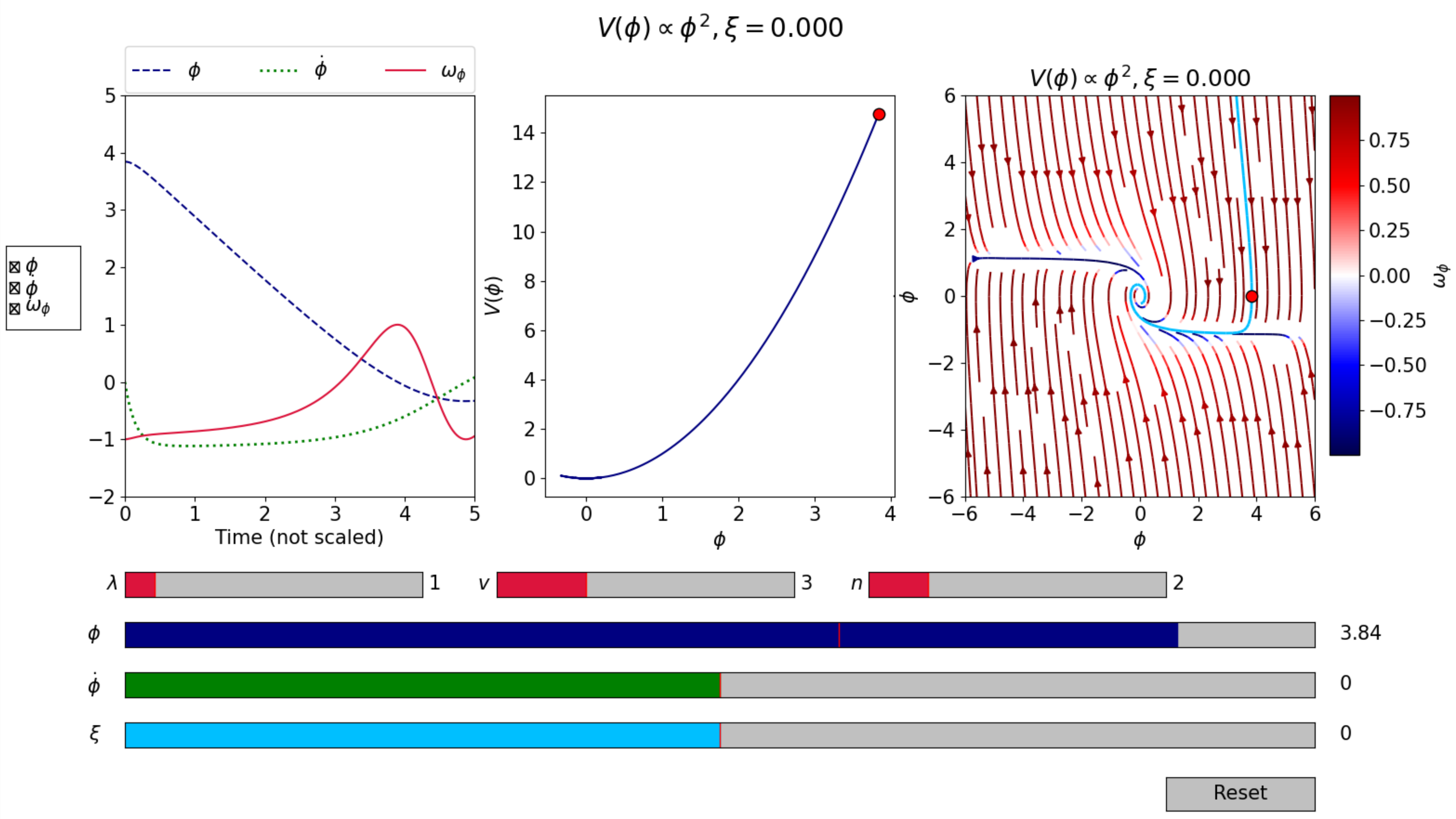
$$\eta = \frac{1}{\kappa^2}\frac{V''}{V} = \frac{1}{\kappa^2}\frac{n(n-1)}{\varphi^2}$$

$$N = -\kappa^2 \int_{\varphi_*}^{\varphi_e} \frac{\varphi}{n} d\varphi = -\frac{\kappa^2}{2n}\left(\varphi_e^2 - \varphi_*^2\right) \longrightarrow \boxed{\varphi_*^2 = \frac{n(4N+n)}{2\kappa^2}}$$

$$\epsilon(\varphi_*) = \frac{n}{n+4N} \qquad \eta(\varphi_*) = \frac{2(n-1)}{n+4N}$$



SOME INTERACTIVITY..



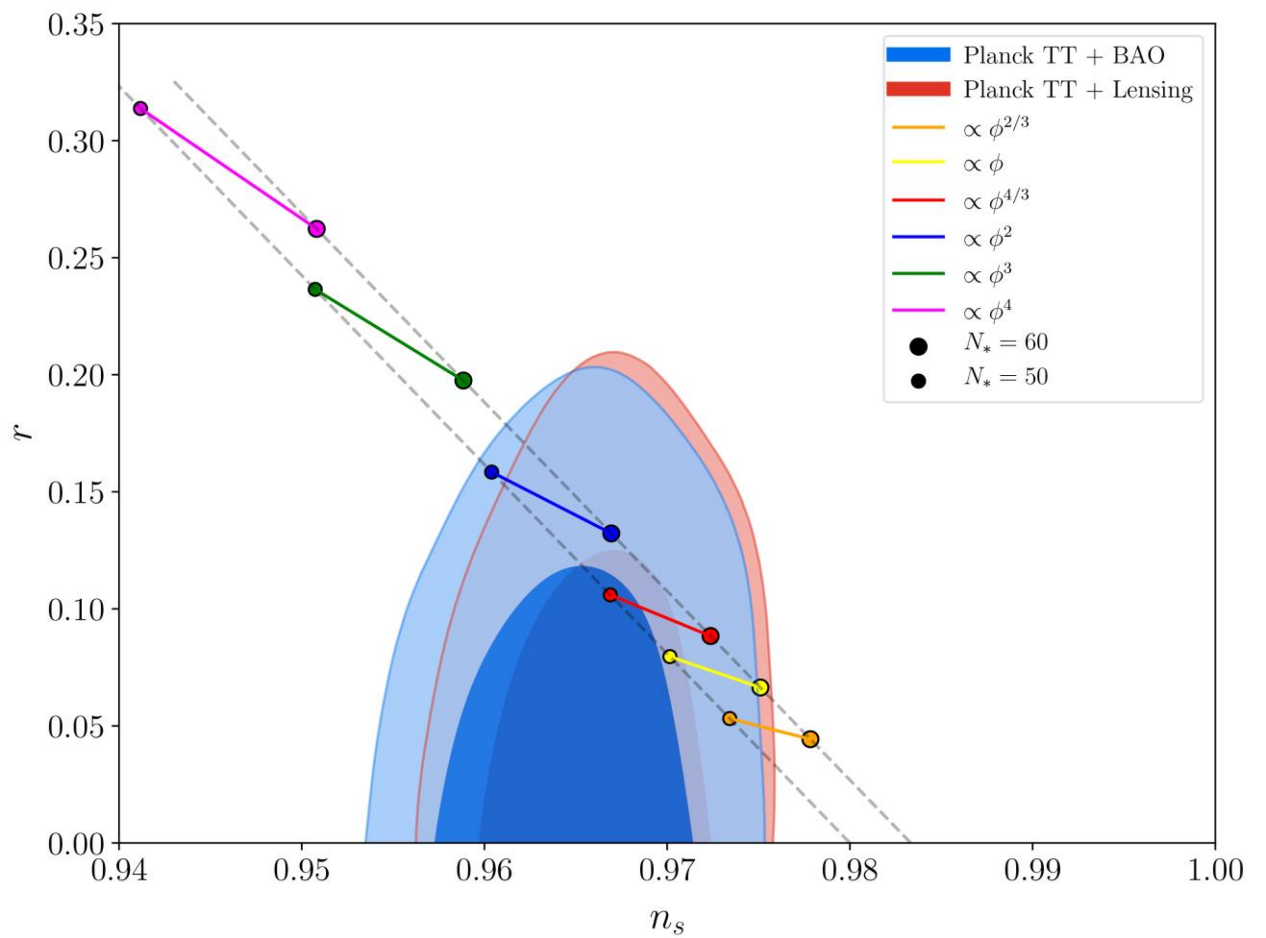
PREDICTIONS OF TODAY'S TOY MODEL

$$V(\varphi) \propto \varphi^n$$

$$\epsilon(\varphi_*) = \frac{n}{n+4N} \quad \eta(\varphi_*) = \frac{2(n-1)}{n+4N}$$

$$r = 16\epsilon(\varphi_*)$$

$$n_s = 1 - 6\epsilon(\varphi_*) + 2\eta(\varphi_*)$$



MODIFYING THEORY

$$S = \int \mathrm{d}^4x\sqrt{|g|}\left\{F(\varphi)R - \frac{1}{2}g^{\mu\nu}\left(\partial_\mu\varphi\right)\left(\partial_\nu\varphi\right) - V(\varphi)\right\} \quad \text{where } F(\varphi) = \frac{1}{2}\left(\frac{1}{\kappa^2} + \xi\varphi^2\right)$$

Einstein Gravity ($\frac{1}{\kappa^2}$)

NMC scalar field ($\xi\varphi^2$)

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2F(\varphi)}\left\{-g_{\mu\nu}\left(\frac{1}{2}(\partial\varphi)^2 + V(\varphi)\right) + 2\nabla_\mu\nabla_\nu F(\varphi) - 2g_{\mu\nu}\Box F(\varphi) + \partial_\mu\varphi\partial_\nu\varphi\right\}$$

$$\Box\varphi + 6F'R - \frac{dV(\varphi)}{d\varphi} = 0$$

<u>In FRW</u>

$$\Box F\varphi = \frac{1}{\sqrt{|g|}}\partial_\mu\left\{\sqrt{|g|}g^{\mu\nu}\partial_\nu F(\varphi)\right\}$$

$$\Box F(\varphi) = -3H\dot{F} - \ddot{F} = -3H\dot{\varphi}F' - \ddot{\varphi}F' - \dot{\varphi}^2F''$$

MODIFYING THEORY

$$S = \int \mathrm{d}^4x \sqrt{|g|} \left\{ F(\varphi)R - \frac{1}{2} g^{\mu\nu} \left(\partial_\mu \varphi\right)\left(\partial_\nu \varphi\right) - V(\varphi) \right\} \quad \text{where } F(\varphi) = \frac{1}{2}\left(\frac{1}{\kappa^2} + \xi\varphi^2\right)$$

Einstein Gravity ($\frac{1}{\kappa^2}$)

NMC scalar field ($\xi\varphi^2$)

$$6F(\phi)H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 6H\dot{F}(\phi)$$

$$4F(\phi)\dot{H} = -\dot{\phi}^2 - \ddot{F}(\phi) + 2H\dot{F}(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} - 6\left(2H^2 + \dot{H}\right)F'(\phi) + V'(\phi) = 0$$

SLOW-ROLL EQUATIONS: JF

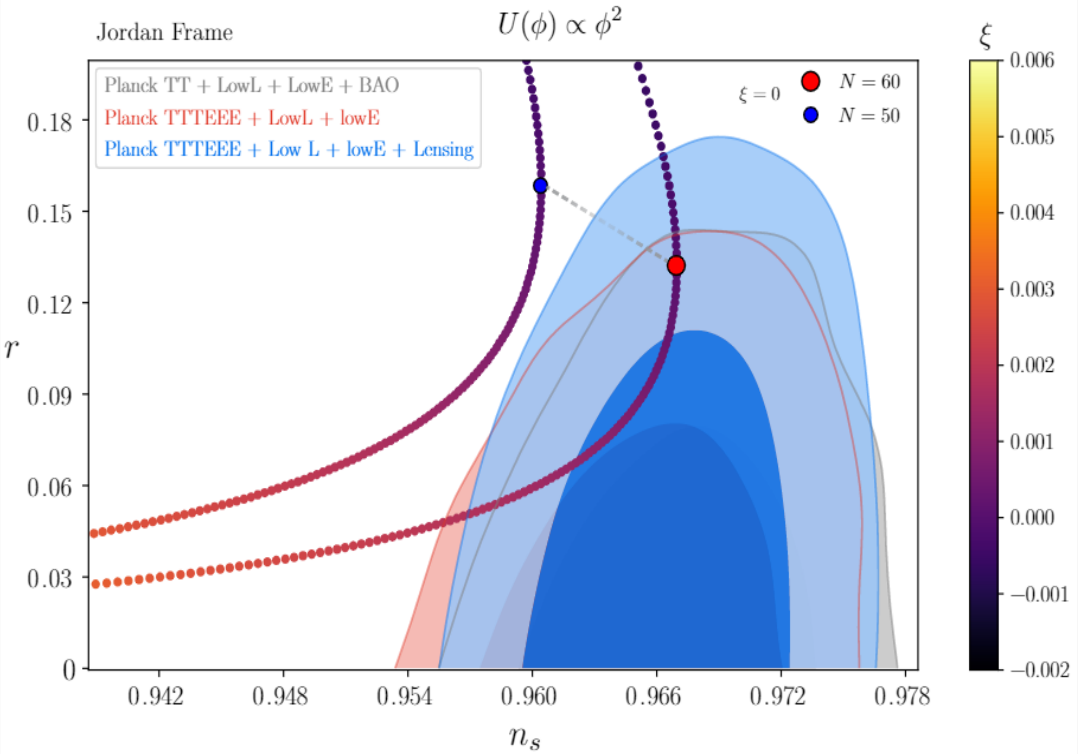
$$\epsilon_H \equiv -\frac{\dot{H}}{H^2}, \quad \eta_H \equiv \frac{\dot{\epsilon}_H}{H\epsilon_H}; \quad \epsilon_F \equiv \frac{\dot{F}}{HF}, \quad \eta_F \equiv \frac{\dot{\epsilon}_F}{H\epsilon_F}$$

$$H^2 \simeq \frac{V}{3F}$$

$$3H\dot{\phi} \simeq 6H^2F' + 3\dot{H}F' - V'$$

$$\dot{H} = \frac{\dot{\phi}}{6HF^2}(V'F - VF') = \frac{\dot{\phi}}{6H}\left(\frac{V}{F}\right)'$$

$$N = \int H dt = \int_{\phi_i}^{\phi_e} \frac{H}{\dot{\phi}} d\phi = \int_{\phi_i}^{\phi_e} \frac{3VF'^2 + 2F}{F(4V'F' - V'F)} d\phi \, .$$



$$r = 8\,(2\epsilon_H + \epsilon_F)$$

$$n_s = 1 - 2\epsilon_H - \epsilon_F - \frac{2\epsilon_H\eta_H + \epsilon_F\eta_F}{2\epsilon_H + \epsilon_F}$$